$\mathrm{Re}_x = (\bar{U}x)/\nu$; Pr, Prandtl criterion; x, y, axial and normal coordinates; r, heat of vapor formation; C_p, specific heat; ψ, stream function. Indices: R, axis; e, evaporator; 0, evaporation surface; t, heat conduction.

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ACTIVATION OF VAPORIZATION CENTERS. I*

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The influence of dissolved gases on the formation of a vapor-bubble nucleus of critical dimensions is considered. A mathematical model is proposed for the subsequent evolution of the nucleus; it gives a correct description of the growth of the vapor bubble up to the instant at which it is detached from the wall.

Even after allowing for the temperature dependence of the surface tension σ and molecular heat of vaporization λ, the theory of heterophase fluctuations [1] leads [2] to values of the degree of superheating required to vaporize liquids roughly twice as great as those measured experimentally. Harvey [3] noted that microscopic gas nuclei might survive in the indentations of rough, unwetted solid surfaces, causing liquids to boil at very slight superheatings. The idea of micronuclei constitutes the basis for the theory of the deactivation of indentations proposed by Holz and Singer and set out in [4]. It follows from [5, 6] that stable gas micronuclei are absent in the case of organic liquids. For these, as well as for liquid metals which wet adjacent solid surfaces almost completely [4], the deactivation theory cannot explain the fact of early boiling. A new physical model was proposed in [2, 7] for the initial stage of phase transformations in liquids; this model may help in explaining the boiling of organic liquids and molten metals. According to [2, 7], complexes of several vapor molecules formed as a result of the superheating of the boundary layer of liquid are adsorbed in indentations on the surface, forming nuclei of greater than critical size. This paper is a continuation of [2, 7] and considers the influence of dissolved gases on the boiling of liquids, as well as formulating a mathematical model for the further evolution of the nucleus.

One of the reasons for the formation of gas micronuclei on a solid surface is the adsorption of gas dissolved in the liquid on surface indentations. Let us consider an indentation of conical shape with a depth z_0 and a base radius r_0. The number N_a of gas molecules adsorbed in the indentation is determined by the adsorption isotherm derived in [7, 8], which has the following form for a conical indentation:

$$N_a = \frac{\gamma g n_s \pi r_0 P \sqrt{r_0^2 + z_0^2}\left[1 - \left(\frac{P}{P_0}\right)^{\beta}\right]\exp\left(\frac{\Psi}{kT}\right)}{\left(1 - \frac{P}{P_0}\right)\left\{1 - \frac{P}{P_0} + \gamma g P\left[1 - \left(\frac{P}{P_0}\right)^{\beta}\right]\exp\left(\frac{\Psi}{kT}\right)\right\}}. \tag{1}$$

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Here $(1/g) = \nu n_S (2\pi m_0 kT)^{1/2}$; $\beta = z_0/d_2$; P and T are the pressure and temperature of the liquid; n_S is the number of adsorption centers on unit area of the surface of the indentation; k is Boltzmann's constant; $\nu \exp[-(\Psi/kT)]$ is the desorption velocity constant [9]; d_2 is the average distance between adsorbed molecules of mass m_0; P_0 is the saturated vapor pressure of the liquid at temperature T; $\Psi = -\Phi(r_0, z_0, d)|_{d={}^1/_2 z_0}$. The potential energy $\Phi(r_0, z_0, d)$ of a molecule adsorbed at a distance d from the tip of the cone may be found by making use of the Mie dispersion-interaction potential $U(h) = Ah^{-n'} - Bh^{-m'}$ ($n' = 9$, $m' = 6$, $h^2 = r^2 + z^2$) on the basis of Eqs. (1) and (2) of [7]. After these calculations we obtain

$$\Phi(r_0, z_0, d) = -\frac{\pi}{12} n_0 B \left[\frac{1}{d^3} \left(1 + fb^2 + \frac{3}{2} w \right) - \frac{fb^2}{r_0^3} \right], \tag{2}$$

where n_0 is the number of molecules in unit volume of the material of the solid wall; $b = r_0^2/(r_0^2 + z_0^2)$; $w = fb + \arctan f$; $f = z_0/r_0$. The coefficient γ from (1) is the probability that gas molecules will collide with the surface of the indentation; it may be found from the kinetic interpretation of the Henry law:

$$P_g = N_s \left(\frac{m'n'kT\varepsilon m_0}{3\pi\mu} \right)^{1/2} \frac{\left(\frac{\varepsilon}{kT} \right)^{s-1}}{(s-1)!} \exp\left(-\frac{\varepsilon}{kT} \right) \equiv N_s F(T), \tag{3}$$

where N_S is the molecular concentration of the dissolved gas; μ is the reduced mass of the liquid – gas system; ε is the energy required to bring a gas molecule out of solution; 2s is the number of quadratic terms in the sum used to represent the energy of the molecules; m' and n' are the power indices of the Mie potential. Allowing for (3) $\gamma = [P_g/NF(T)]$ (N is the total number of molecules per unit volume of the solution).

In an unwetted indentation on a rough surface, the pressure P_{b0} of the adsorbed gas (or gas – vapor mixture) is smaller than the pressure of the liquid by an amount depending on σ, the radius of the meniscus r_1, the wetting angle Θ_1, and the aperture angle of the indentation $2\alpha_1$ (see [4]), i. e., $P_{b0} = P - [2\sigma|\cos(\Theta_1 - \alpha_1)|/r_1]$. The height z_1 (reckoned from the vertex of the conical depression) to which the depression is filled with gas may be found from the relation $({}^1/_3)\pi(r_0/z_0)^2 z_1^3 = (N_a/L_0)(P_1 V_1 T/P_{b0} T_1)$, where $L_0 = 2.7 \cdot 10^{19}$ is the number of molecules occurring under normal conditions (P_1, T_1) in a volume of gas $V_1 = 1$ cm^3 (Loschmidt number).

On heating the boundary layer of liquid from T_0 to the saturation temperature T_S^* corresponding to the pressure P, the adsorbing capacity of the indentation, i.e., N_a, diminishes. This is taken into account by substituting $T = T_S^*$ into N_a and γ. However, in addition to this, as T rises the solubility of the gas will diminish, and the excess of gas will pass into the space above the liquid to the relative extent $0 < \varphi < 1$ being trapped in the indentations to the extent $1 - \varphi$, so increasing the number and volume of the gas bubbles already present in these. (An analogous situation arises on the walls of a bottle containing alcohol.) This not only eases the boiling of the liquids, but also increases the intensity of heat transfer by 50% (on account of the growth of the vaporization centers) when boiling under conditions of free convection [10, 11]. The increment to $N_a(T_S^*)$ associated with the reduction in the solubility of the gas (on the assumption that P_g = constant) is

$$\Delta N_a = \frac{(1-\varphi) P_g}{N_{ind}} \left[\frac{\delta}{F(T_0)} - \int_0^\delta \frac{dx}{F[T(x, t_0)]} \right],$$
$$T(x, t_0) = T_0 + \Theta(x, t_0). \tag{4}$$

Here N_{ind} is the number of indentations per unit area of the rough solid surface, δ is the thickness of the layer of liquid, while $\Theta(x, t)$ is given by (8). In addition to this, over the period t_0 required for heating the boundary layer of liquid from T_0 to T_S^*, a certain number molecules ΔN_b will pass into the micronucleus through its interface with the liquid as a result of evaporation. This will be established below.

The temperature field in the liquid depends on the intensity of the convective currents excited in the liquid. For natural turbulent convection we may introduce a coefficient of effective thermal conductivity [12]:

$$\lambda^*(x, \Theta) = 0.073 \lambda_0 \left(\frac{\nu}{a_0} \right)^{0.55} \left(\frac{\beta g}{\nu^2} \right)^{1/3} x\Theta^{1/3} \equiv Mx\Theta^{1/3}, \tag{5}$$

if the x axis is directed along the normal to the lower solid heating wall. The parameters in Eq. (5) were defined in [12]. Let us suppose that at $x = \delta$ the temperature of the liquid is held constant and equal to T_0

(in °K), i.e., we shall consider an underheated liquid, while at x = 0 a constant thermal flux q_0 is supplied to the latter. The solution of the nonlinear problem

$$\frac{\partial\Theta}{\partial t}=\frac{M}{c\rho}\frac{\partial}{\partial x}\left(x\Theta^{1/3}\frac{\partial\Theta}{\partial x}\right),\quad \Theta(x,\ t)=T(x,\ t)-T_0,$$

$$-\lambda\left(\frac{\partial\Theta}{\partial x}\right)\Big|_{x=0}=q_0,\quad \Theta(\delta,\ t)=0,\quad \Theta(x,\ 0)=0 \tag{6}$$

may be obtained if $\lambda*(x,\ \Theta)$ is expressed in the form $(^1/_2)M\delta\Theta_S^{1/3}-Mx\Omega(\Theta)$, where $\Theta_S=T_S^*-T_0$ and $\Omega(\Theta)$ is a nonlinear function. In order to linearize the equation, in the term $Mx\Omega(\Theta)$ we may use the function Θ_0 derived from a solution of the problem with $\Omega(\Theta)=0$. Applying a Laplace transformation with respect to time we find that

$$\Theta_0(x,\ t)=\frac{q_0}{\lambda}\left\{\frac{2}{3}(\delta-x)-\frac{8\delta}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\times\right.$$

$$\left.\times\exp\left[-\frac{a(2n+1)\pi^2}{4\delta^2}t\right]\sin\left[\frac{(2n+1)(\delta-x)}{2\delta}\pi\right]\right\},\ a=\frac{\lambda}{c\rho}=\frac{M\delta\Theta_s^{1/3}}{2c\rho}. \tag{7}$$

Substituting Θ_0 into $\Omega(\Theta)$ we arrive at a linear equation with a source $H(x,\ t)=(M/c\rho)(\partial/\partial x)[x\Omega(\Theta_0)(\partial\Theta_0/\partial x)]$. The solution subject to the boundary conditions in (6) is

$$\Theta(x,\ t)=\int_0^t dt\int_0^\delta H(x_0,\ \tau)G(x_0;\ x,\ t-\tau)\,dx_0+\frac{q_0}{c\rho}\int_0^t d\tau G(0;\ x,\ t-\tau), \tag{8}$$

where G is the Green's function of a mixed boundary problem of the first and second kinds. For small time values

$$G(x_0;\ x,\ t)=\frac{1}{2\sqrt{\pi at}}\sum_{n=-\infty}^{\infty}(-1)^n\left\{\exp\left[-\frac{(x-x^*)}{4at}\right]-\exp\left[-\frac{(x+x^*)}{4at}\right]\right\}. \tag{9}$$

Here $x^*=x_0+(2n-1)\delta$, while for large $F_0=(at/\delta^2)$

$$G(x_0;\ x,\ t)=\frac{2}{\delta}\sum_{n=0}^{\infty}(-1)^n\exp\left[-\frac{(2n+1)^2\pi^2}{4}\mathrm{Fo}\right]\sin\frac{(2n+1)\pi x_0}{2\delta}\sin\frac{(2n+1)\pi x}{2\delta}. \tag{10}$$

An analogous solution may be obtained for the problem in which the thermal flux q_1 is specified on the boundary $x=\delta$, and also for that in which q_0 and q_1 are specified in the form of arbitrary functions of t. The evolution of the bubble after separation from the wall depends very considerably on the form of the boundary conditions.

From Eq. (8) we find the value of t_0 for which $T_0+\Theta(0,\ t_0)=T_S^*$. Thus,

$$\Delta N_b=\pi z_1^2\left(\frac{r_0}{z_0}\right)^2\int_0^{t_0}dt\frac{4Nr\eta_0(P_{b0})}{m_1\sqrt{\pi}(kT_*)^{3/2}}\int_\lambda^\infty\varepsilon\exp\left(-\frac{\varepsilon}{kT_*}\right)d\varepsilon, \tag{11}$$

where a Maxwell function is used as the energy distribution function of the molecules in the liquid (the number per unit volume being N); $T*(t)=T_0+\Theta(0,\ t)$; r is the reflection coefficient; $\eta_0(P_{b0})$ is the condensation coefficient; λ is the molecular heat of vaporization; m_1 is the mass of the liquid molecule. After certain calculations we obtain

$$\Delta N_b(t_0)=4\sqrt{\pi}z_1^2Nr\eta_0(P_{b0})\frac{1}{m_1}\left(\frac{r_0}{z_0}\right)^2\sqrt{k}\int_0^{t_0}\sqrt{T_*}\left(\frac{\lambda}{kT_*}+1\right)\exp\left(-\frac{\lambda}{kT_*}\right)dt. \tag{12}$$

Thus, at the instant of time t_0 in an indentation of depth z_0 we find $N_0(z_0,\ t_0)=N_a(T_S^*)+\Delta N_a+\Delta N_b$ gas and vapor molecules occupying a height $z_2=[(3/\pi)(z_0/r_0)^2(P_1V_1T_SN_0/L_0T_1P_{b0})]^{1/3}$, on the assumption that Θ_1 and r_1 have not varied up to the moment t_0. Since the depths of the indentations obey a Gauss distribution with a dispersion σ_0^2, the number of potential centers of vaporization activated at the instant t_0 is determined by the relation

$$n = \frac{N_{\text{ind}}}{\sigma_0 \sqrt{2\pi}} \int\limits_{z_0^*}^{z_0^{**}} \exp\left(-\frac{z_0^2}{2\sigma_0^2}\right) dz_0 = \frac{1}{2} N_{\text{ind}} \left[\Phi\left(\frac{z_0^{**}}{\sigma_0\sqrt{2}}\right) - \Phi\left(\frac{z_0^{*}}{\sigma_0\sqrt{2}}\right)\right], \tag{13}$$

where $\Phi(x)$ is the probability integral, while $z_0^{**} = f(z_2)$ and $z_0^* = f(z_{cr})$ are determined from the expressions $z_2 = [Dz_0^2 T_s^* N_0(z_0, t_0)]^{1/3}$ (for $z_0 = z_0^{**}$) and $z_{cr} = [Dz_0^2 T_s^* N_{cr}(z_0, t_0)]^{1/3}$ (for $z_0 = z_0^*$). Here $D = (3/\pi)(P_1 V_1/r_0^2 L_0 T_1 P_{b0})$, while N_{cr} is the number of gas and vapor molecules in a nucleus of critical size. For $t_* > t_0$ at which $\Theta(0, t_*) = \Theta_s + \Delta\Theta$, i.e., for a superheating $\Delta\Theta$ of the liquid, the number of activated vaporization centers is obtained by making the substitutions $t_0 \to t_*$ and $T_s^* \to T_s^* + \Delta\Theta$. Since for large γ the role of the dissolved gases in the formation of complexes of $N_a + \Delta N_a$ molecules is predominant, even slight superheatings of the liquid will lead to intense vaporization, as may readily be deduced from Eq. (11). This fact is of fundamental importance for the boiling of liquid metals in nuclear reactors [4,10].

Let us write down the system of equations determining the subsequent growth of a gas — vapor bubble within an indentation of the rough surface. Owing to the small dimensions of the indentation we may assume that the temperature in the bubble varies in accordance with the law $T_b(t) = T_*(t)$ for $t > t_0$. The relationship between the parameters of state of the vapor in the bubble are given in simplified form by the equation

$$P_b(t) = \frac{Rm(t) T_b(t)}{\Psi_1 V(t)}, \quad V(t) = \frac{1}{3}\pi \left(\frac{r_0}{z_0}\right)^2 z^3(t), \quad z_2 \leqslant z \leqslant z_0 + r_0, \tag{14}$$

where Ψ_1 is the form factor; R is the specific gas content. (This relationship was established in [13] on the basis of the results of [14].) Using the Langmuir — Hertz — Knudsen formula, we may find the change in the mass of vapor in the bubble:

$$\frac{dm}{dt} = \pi z^2(t) \eta \left(\frac{r_0}{z_0}\right)^2 (P_s - P_b) \left(\frac{M_0}{2\pi R T_*}\right)^{1/2}, \tag{15}$$

where P_s is the saturated vapor pressure of the liquid; M_0 is the molecular weight of the vapor; $\eta(P_b)$ is the effective vaporization coefficient (also called the coefficient of condensation [15]). The quantity η was listed in [15] for a number of substances. The boundary layer of liquid and also that of the vapor bubbles at the instants of time preceding the onset of boiling are superheated relative to T_s^*. Each value of $T_*(t)$ corresponds to its own saturation pressure P_s. The $P_s(T_*)$ relationship is given by the Clasius — Clapeyron formula $[d \ln P_s/dT_*] = (L/RT_*^2)$ or the Kirchhoff — Rankine — Dupré equation

$$\ln P_s = u + \left(\frac{\Delta C_p}{R}\right) \ln T_* - \frac{L}{RT_*}, \quad u = \text{const}. \tag{16}$$

Here L is the latent heat of vaporization of 1 g-mole of liquid; ΔC_p is the difference between the specific heats of the vapor and condensate. In order to close the system of equations we must establish a relationship between the vapor pressure in the bubble P_b, the pressure of the liquid P, and the dimensions of the bubble, i.e., we must write down the analog of the Rayleigh equation for a bubble of conical shape. Let us neglect the forces of surface friction associated with the motion of the liquid inside the indentation, i.e., consider this motion as spherically symmetrical. From the Navier — Stokes equation and the equation of continuity, by analogy with [16], we find that

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{1}{\rho_0}\frac{\partial P}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r}\right), \quad u = \dot{z}\left(\frac{z}{r}\right)^2, \ \dot{z} \equiv \frac{dz}{dt}.$$

Integrating the equation for u with respect to r between z and δ, making use of the Laplace relationship [20] $P(z) = P_b - (2\sigma/z) + (4/3)\rho_0\nu[(\partial u/\partial r) - (u/r)]|_{r=z}$, and assuming that $P(\delta) = P$, we obtain an analog of the Rayleigh equation:

$$(\ddot{z}z + 2\dot{z}^2)(1 - \alpha z) - \frac{1}{2}\dot{z}^2(1 - \alpha^4 z^4) - \frac{2}{3}\nu\frac{\dot{z}}{z}(1 - \alpha^3 z^3) = \frac{1}{\rho_0}\left(P_b - P - \frac{2\sigma}{z} - 4\nu\rho_0\frac{\dot{z}}{z}\right), \tag{17}$$

in which $\alpha = \delta^{-1}$; $z_2 \leq z \leq z_0 + r_0$; ν is the kinematic viscosity of the liquid; ρ_0 is the mass of unit volume of the liquid; δ is the thickness of the liquid layer.

By solving the system of equations (6) and (14)-(17) we may find the laws governing the changes in $m(t)$, $P_s(t)$, $P_b(t)$, $T_*(t)$, $z(t)$ and the value of t_1 for which $z(t_1) = z_0 + r_0$. The initial conditions for these equations take the form $P_b(t_0) = P_{b0}$, $m(t_0) = m_0 N_0$, $T_b(t_0) = T_*(t_0)$, $z(t_0) = z_2$, $P_s(t_0) = P_s(T_s^*)$.

Let us consider the following stage in the evolution of the bubble from a hemisphere of radius r_0 attached to the plane of the wall to a sphere with a separation radius R_{sep}. As before, the decisive factor in the development of the bubble is the superheating of the liquid and the vapor relative to T_S^*, as a result of which the vapor pressure in the bubble P_b exceeds the external pressure of the liquid P. (It follows from [17] that even at a temperature of 120°C the P_S of the water vapour is almost twice atmospheric pressure. The $P_S(T_S)$ relationship is also given in [18].) Let us assume that

$$P_{\text{b}}(t)=\frac{3}{4}\frac{Rm(t)\,T_{\text{b}}(t)}{\pi\Psi_2 r^3(t)},\ \Psi_2=\frac{1}{2}\left[1+\frac{1}{2}\cos\Theta_1(2+\sin^2\Theta_1)\right]+\frac{1}{4}\frac{\delta_1}{r}\sin^2\Theta_1. \tag{18}$$

The quantities in the form factor Ψ_2 are listed in [13]. The relationship between the saturation pressure and the vapor temperature follows from the equation

$$\ln P_s=u+\left(\frac{\Delta C_p}{R}\right)\ln T_{\text{b}}(t)-\frac{L}{RT_{\text{b}}(t)} \tag{19}$$

on the assumption that $P_S(T_S) = P_S(T_b)$. The rate of evaporation of the molecules inside the bubble depends on its wall temperature $T_f(t)$. Since $T_f(t)$ is different on different parts of the surface $S(t) = 4\pi r^2(t)$ of the bubble, we have

$$\frac{dm}{dt}=\iint\limits_{S(t)}q(t)\,dS,\ q(t)=\eta(P_{\text{b}})(P_s-P_{\text{b}})\left(\frac{M_0}{2\pi RT_f}\right)^{1/2}. \tag{20}$$

The transient spatially inhomogeneous temperature distribution $T_b(r, t)$ in the growing bubble should be determined from the heat-conduction equation for an expanding spherical region. Approximately we have $T_b = T_b(t)$ where $T_b(t)$ is found from the heat balance associated with the heat transfer (transfer coefficient k_0) between the vapor and the surrounding liquid:

$$\iint\limits_{S(t)}k_0(T_f-T_{\text{b}})\,dS=\frac{4}{3}\pi r^3(t)\,C_{\text{b}}\frac{dT_{\text{b}}}{dt}. \tag{21}$$

Here C_b is the volumetric specific heat of the vapor. If we use the gravimetric specific heat, then on the right-hand side of (21) we must introduce the vapor density $\rho_b(t) = (^3/_4)[m(t)/\pi r^3(t)]$. Equation (21) enables us to make an automatic allowance for the temperature jumps between the liquid and the vapor [11, 19].

Neglecting effects associated with the absence of spherical symmetry during the motion of the liquid surrounding the bubble, and also neglecting the effect of the finite thickness of the layer of liquid, i.e., putting $\alpha = 0$ in (17), we find, in the same way as that employed in deriving Eq. (17), that the relationship between the change in the radius of the bubble growing on the wall and the pressure drop $\Delta P = P_b - P$ is given by the equation

$$r\frac{d^2r}{dt^2}+\frac{3}{2}\left(\frac{dr}{dt}\right)^2+\varphi(\nu,\ r,\ \dot r)=\left(\Delta P-\frac{2\sigma}{r}\right)\frac{1}{\rho_0},$$

$$\varphi(\nu,\ r,\ \dot r)=4\nu\frac{\dot r}{r}-\frac{2}{3}\nu\frac{\dot r}{r}=\frac{10}{3}\nu\frac{\dot r}{r}. \tag{22}$$

It is usually considered that in the Rayleigh equation $\varphi(\nu, r, \dot{r}) = 4\nu(\dot{r}/r)$, i.e., only the difference between the pressure and the normal forces in liquids due to viscosity effects is taken into account. It is nevertheless quite clear that in order to take a proper account of viscosity we must also start from the equation of motion in the Navier – Stokes for m and not the Euler form. This has the effect that $\varphi(\nu, r, \dot{r}) = (^{10}/_3)\nu(\dot{r}/r)$.

Over the range $r_0 \to R_{sep}$ the growth of the bubble is determined by the three-dimensional temperature field derived from the equation

$$\frac{\partial T}{\partial t}=a\frac{\partial^2T}{\partial x^2}+a_0\left(\frac{\partial^2T}{\partial y^2}+\frac{\partial^2T}{\partial z^2}\right)-H_1(x,\ t),\ a_0=\frac{\lambda_0}{c\rho}, \tag{23}$$

$$H_1(x,\ t)=\frac{M}{c\rho}\frac{\partial}{\partial x}\left[x\Omega(\Theta_0+T_0)\frac{\partial\Theta_0}{\partial x}\right]$$

with the boundary condition at the surface of the bubble

$$\left[-\lambda_0 4\pi r_{\text{b}}^2\frac{\partial T}{\partial r}+\frac{\lambda}{m_0}\iint\limits_{S(t)}\eta(P_s-P_{\text{b}})\left(\frac{M_0}{2\pi RT_f}\right)^{1/2}dS+\iint\limits_{S(t)}k_0(T_f-T_{\text{b}})\,dS\right]_{r=r_{\text{b}}}=0, \tag{24}$$

where $r_b(t)$ is the radius of the bubble; $T_f = T(r, t)|_{r=r_b}$; λ_0 is the thermal conductivity of the liquid. As the initial condition for (23) we use the function $T_1 = \Theta_1 + T_0$ obtained from a solution of (23) at the instant $t = t_1$, i.e., without allowing for the boundary condition on $S(t)$. The boundary condition at $x = 0$ is the same for (23) as it is in the case of (6). As initial conditions for the remaining unknowns we use their values obtained from a solution of the previous system of equations at $t = t_1$.

Equations (18)-(23) allow for the evaporation of the microlayer, not only under the bubble, but also over its whole surface. In this way the equations strictly and automatically incorporate the now generally accepted hypothesis of the microlayer (see bibliography to [21]) which received experimental confirmation in [22]. Other approaches and growth models for vapor bubbles on solid walls appear in [13, 20, 23, 24]. For volumetric boiling the same question was treated in [19].

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